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RESEARCH MEMORANDUM

EMPIRICAL MODE CONSTANTS FOR CALCULATING FREQUENCIES OF AXIAL-FLOW COMPRESSOR BLADES

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EMPIRICAL MODE CONSTANTS FOR CALCULATING FREQUENCIES OF AXIAL-FLOW COMPRESSOR BLADES

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SUMMARY

Experimentally determined mode constants for computing the natural frequencies of vibration of axial-flow compressor blades in the first three modes of bending and torsion are presented. The ratios of depth-to-chord and length-to-chord for which these modes constitute the principal vibrations occurring were also determined for blades of uniform rectangular section to find approximate limits for the application of these mode constants to actual blades. A comparison of experimentally determined frequencies of a second group of blades with frequencies computed using these mode constants showed that the computed values were correct within 10 percent.

INTRODUCTION

Approximate prediction of the vibrational characteristics of axial-flow compressors is desirable in order that critical speeds may be minimized by design rather than by development of the compressor itself. Such predictions require the determination of the frequencies of the blading. Several methods, such as that of Myklestad (reference 1), are available for predicting such frequencies but most of them are lengthy and involve a detailed knowledge of the geometry of each individual blade. Myklestad's method is a trial-and-error calculation and the computation time is shortened if approximate values of frequency are known in advance. A method of computing the bending frequencies, which is fairly rapid but is still dependent on the geometry of the particular blade being studied, is given in reference 2 (pp. 386-387). In addition, the geometry of compressor blades so far investigated is in the region in which this method is least accurate.

There is a possibility that because the variation in geometry of compressor blades is limited, mode constants can be determined that will give approximate frequency values for any compressor blade. Such constants were determined at the NACA Cleveland laboratory for a number of blades, averaged, and used to compute frequencies of another group of blades

SYMBOLS

The following symbols are used:

A area, (sq in.)

a_m bending-mode constant of a uniform beam for mode m

b chord, (in.)

E Young's modulus, (lb/sq in.)

f frequency, (cps)

G modulus of rigidity, (lb/sq in.)

I moment of inertia of cross section in bending direction, (in.^{4})

J polar moment of inertia, (in.^{4})

K torsional stiffness factor, (in.^{4})

l length of blade or beam, (in.)

α_m bending-mode constant of a blade for mode m

β_m torsional-mode constant of a blade for mode m

ρ mass density, (lb sec^2/in.^{4})

Subscript:

o fixed end of blade

PROCEDURE AND RESULTS

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The frequency of bending vibration of a uniform beam can be expressed as

$$f = \frac{a_m}{l^2}\sqrt{\frac{EI}{\rho A}} \tag{1}$$

Timoshenko (reference 2, pp. 376-382) has shown that the frequency of bending vibration of any beam can be expressed as

$$f = \frac{\alpha_m}{l^2}\sqrt{\frac{EI_o}{\rho A_o}} \tag{2}$$

The value of bending-mode constant α_m in equation (2) is a function of the geometry of the beam as well as the mode shape. If, however, geometry is restricted to a given range, values of α_m can be found that are approximately correct for any beam within this range.

Similarly, the frequency of torsional vibration can be found from the equation

$$f = \frac{\beta_m}{l}\sqrt{\frac{GK_o}{\rho J_o}} \tag{3}$$

where torsional-mode constant β_m varies in a manner similar to α_m.

Roark (reference 3) gives the following expression for torsional stiffness factor K for airfoil sections:

$$K = \frac{4I}{1 + \frac{16I}{Ab^2}} \tag{4}$$

If the airfoil is thin, that is, if I is very small as compared with Ab^2, equation (4) may be reduced to

$$K = 4I \tag{5}$$

A number of compressor blades of known dimensions and geometry were mounted by tightly clamping the base and were excited by means of a 500-watt electronic exciter. Modes were determined by means of sand patterns and the frequencies of the various modes of each blade were recorded. From these data, the values of α_m and β_m were computed for the first, second, and third modes for each blade. The arithmetic-average values of α_m and β_m for each mode were then found. In no case did the average value differ from any individual value by more than 8 percent.

The frequencies of another group of compressor blades were computed using these values of mode constant and were found to agree with experimental values within 10 percent.

The values of mode constant obtained are as follows:

Mode	α_m	β_m
1	0.617	0.275
2	3.14	.672
3	7.89	1.28

The range of geometry covered by the blades studied is shown in figure 1 by means of nondimensional plots of the variations of area A, moment of inertia I, polar moment of inertia J, and torsional stiffness K along the length of the blade.

Blades that were short with respect to their chords did not behave purely as beams but more nearly like plates. In order to determine the limits of beam behavior, a number of uniform beams of various ratios of depth-to-chord and length-to-chord were studied by exciting them at their various natural frequencies and comparing frequencies and mode shapes with those predicted by beam theory. The ranges of these ratios for which the specimen behaved primarily as a plate, as a beam, and the transition area between these two regions are shown in figure 2. Although this plot is for uniform rectangular beams, it is approximately correct for blades if mean values of chord and depth are used.

DISCUSSION

The approximate computation of compressor-blade-vibration frequencies is facilitated by use of experimentally determined mode constants. The accuracy of such computations depends on two factors: first, the error caused by difference in geometry between the blades being computed and that of the blades used in this study; second, errors introduced by differences in clamping between the tight clamping used and the clamping present in the compressor. The clamping of blades mounted in a rotor will vary from blade to blade and, in general, frequencies of blades in a rotor have been found to be lower than corresponding frequencies of the same blade mounted in a tight clamp.

Where more accurate calculations are desired, values of frequency obtained by using the mode constants presented herein are of value in locating the frequency ranges to be investigated by methods such as Myklestad's.

SUMMARY OF RESULTS

Mode constants for the first three bending and torsional modes of vibration were determined experimentally for a group of compressor blades. A comparison of experimentally determined frequencies of a second group of blades with frequencies computed using these mode constants showed that the computed values were correct within 10 percent. The approximate limiting ratios of depth to chord and length to chord below which these constants could not be used to compute the natural frequencies were also found experimentally.

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REFERENCES

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2. Timoshenko, S.: Vibration Problems in Engineering. D. Van Nostrand Co., Inc. (New York), 2d ed., 1937, pp. 386-387; pp. 376-382.

3. Roark, Raymond J.: Formulas for Stress and Strain. McGraw-Hill Book Co., Inc., 2d ed., 1943, p. 171.

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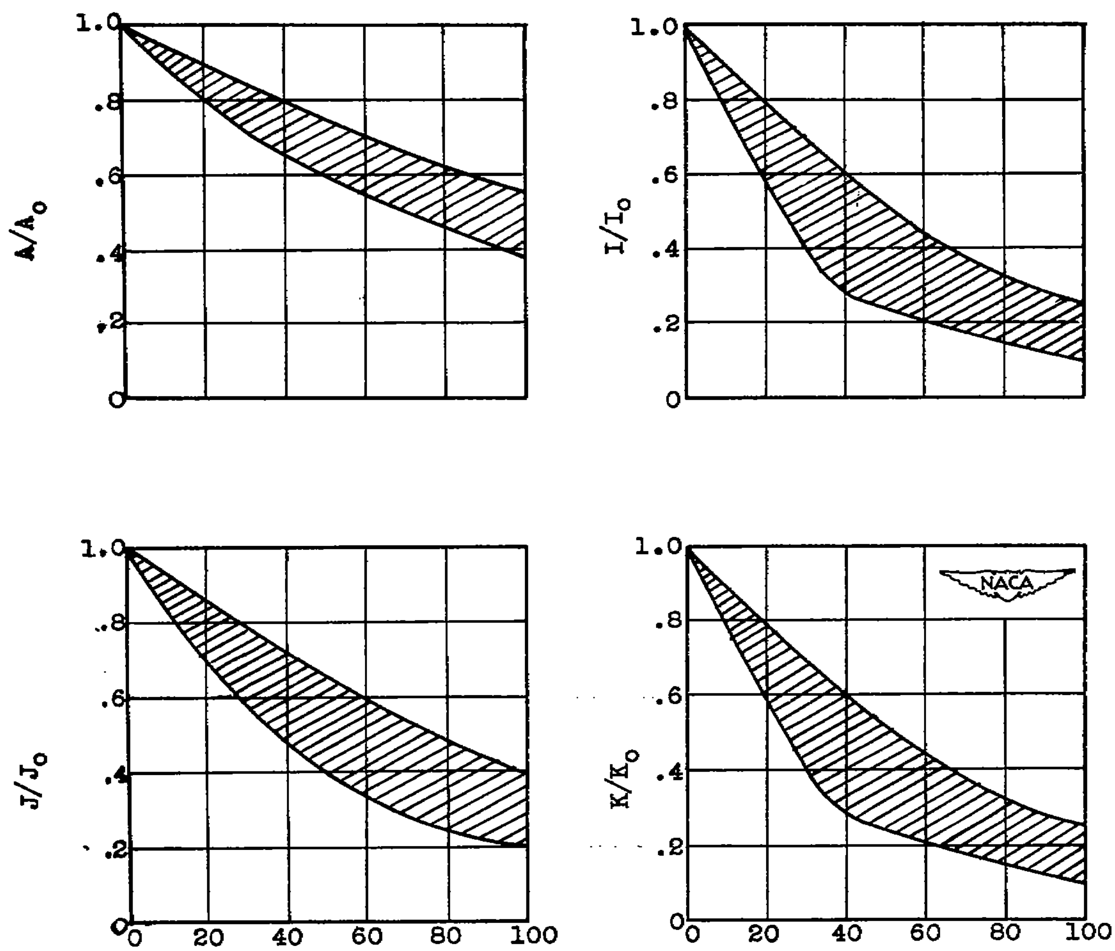

Length, percent

Figure 1. - Geometric range of blades investigated in terms of variations of area A, moment of inertia I, polar moment of inertia J, and torsional stiffness K along length of blade. Subscript o denotes value at fixed end of blade.

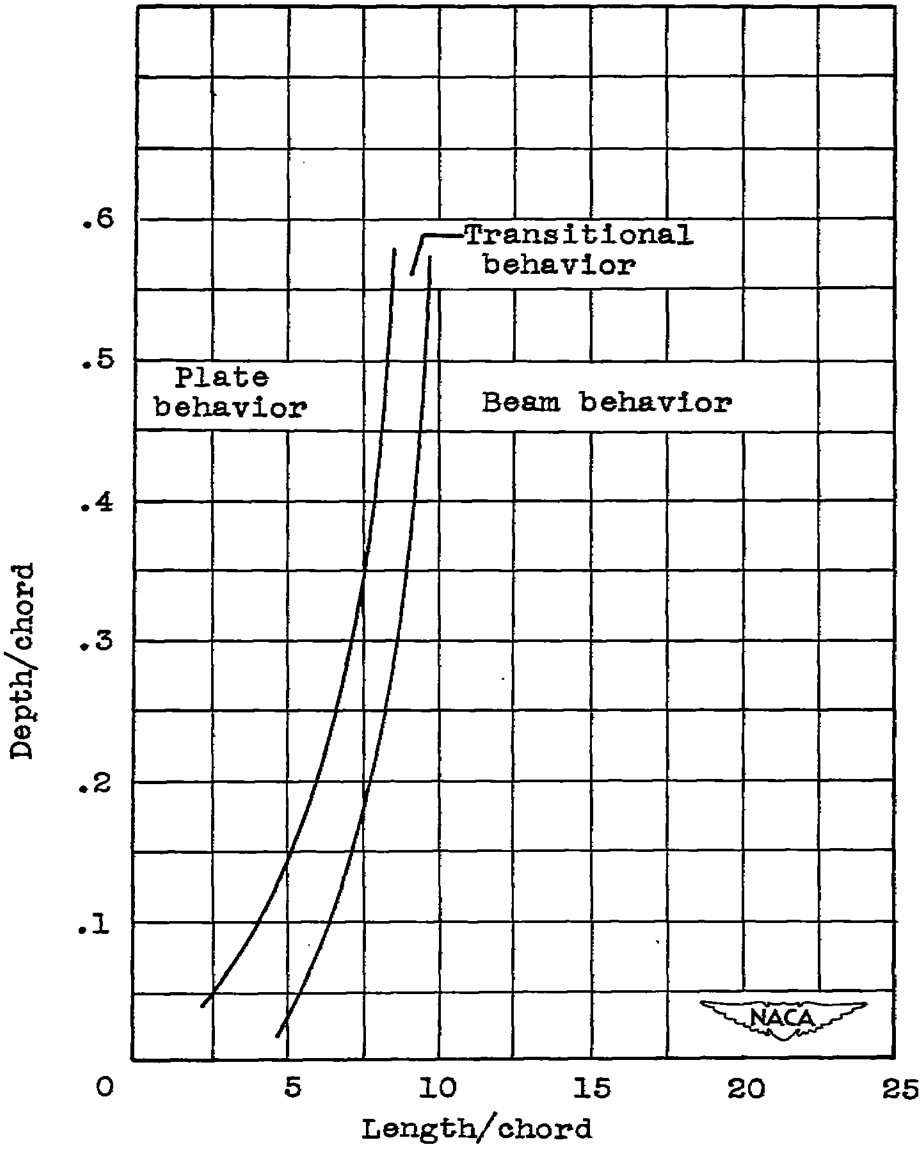

Figure 2. - Experimentally determined regions of plate behavior, beam behavior, and transitional behavior for uniform rectangular blade.